2022 VCE Further Mathematics 1 external assessment report

General comments

Students generally found questions accessible in the Further Mathematics 1 examination in 2022. They found some questions involving the application of the key skills and key knowledge from the study design challenging, such as:

- Questions 1 and 16 from Data analysis
- Questions 17, 21 and 24 from Recursion and financial modelling
- Questions 6, 7 and 8 from Module 1, Matrices
- Questions 3, 4 and 8 from Module 2, Networks and decision mathematics
- Questions 5, 6 and 8 from Module 3, Geometry and measurement
- Questions 6 and 8 from Module 4, Graphs and relations.

Specific information

The tables below indicate the percentage of students who chose each option. The correct answers are indicated by shading.

The statistics in this report may be subject to rounding resulting in a total more or less than 100 per cent.

Section A – Core

In 2022, the Core section comprised two components: Data analysis (Questions 1–16) and Recursion and financial modelling (Questions 17–24). Grey shading indicates the correct answer.

Question	Correct answer	% A	% B	% C	% D	% E
1	C	1	1	36	8	54
2	B	7	85	7	1	0
3	D	4	5	13	55	23
4	E	4	3	5	14	74
5	A	76	6	8	7	3
6	D	5	7	10	75	2
7	A	68	5	10	15	2
8	D	28	7	6	56	3
9	E	10	5	8	10	67

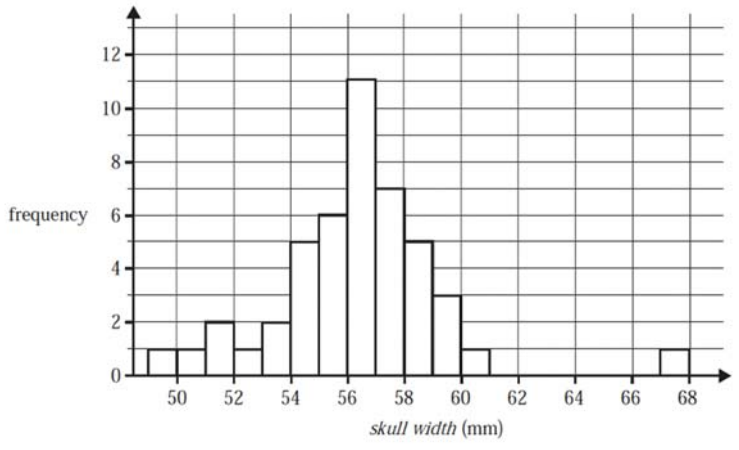
Question	Correct answer	% A	% B	% C	% D	% E
10	C	34	2	56	7	1
11	B	4	86	6	2	2
12	B	4	58	13	13	12
13	B	8	51	20	14	7
14	C	7	24	48	13	7
15	B	6	66	14	7	7
16	A	40	24	6	7	22
17	D	5	32	13	44	6
18	C	8	9	78	3	1
19	A	47	15	12	14	10
20	A	65	6	8	15	5
21	D	3	14	45	33	4
22	C	4	4	58	11	23
23	D	8	15	10	49	18
24	B	7	32	31	16	13

Data analysis

Students generally answered the questions in the Data section very well.

Question 1

The histogram below displays the distribution of *skull width*, in millimetres, for 46 female possums.



Data: adapted from DB Lindenmayer et al., 'Morphological variation among populations of the mountain brushtail possum, *Trichosurus caninus* Ogilby (Phalangeridae: Marsupialia)', *Australian Journal of Zoology*, 43(5), 1995, p. 453

The centre of this distribution is in the group 56–57. Disregarding the possible outlier, we can see that the graph is more spread to the left of the centre than it is to the right. That is, the lower tail is longer than the

upper tail. Therefore, we say that the shape of the distribution is negatively skewed (skewed to the left), with a possible outlier.

Question 14

This question required the calculation of the coefficient of determination, r^2.

Given $r = 0.963$

$r^2 = 0.9273\ldots$ or 92.7%

The percentage of variation **not** explained is 100% - 92.736…% = 7.263…%. The closest is 7.3%.

Question 16

To correct for seasonality the actual value must be divided by the seasonal index.

Dividing by 1.25 is equivalent to reducing by 20%.

From $D = \frac{A}{I} = \frac{A}{1.25} = 0.8A$, as $\frac{1}{1.25} = 0.8$

Recursion and financial modelling

Students did not score as highly on questions involving the use of recurrence relations or the finance solver (Questions 17, 19, 21, 23 and 24).

Question 17

Using the given recurrence relation:

$R_0 = 2$, $R_{n+1} = 2 - R_n$

$R_1 = 2 - 2 = 0$

$R_2 = 2 - 0 = 2$

Question 19

Two steps are required.

Step 1: Determine the annual interest rate.

$R = 1 + \frac{r}{1200}$ $1.003 = 1 + \frac{r}{1200}$ $r = 3.6\%$

Step 2: Use Finance Solver

N = SOLVE = 300 (300 ÷ 12 = 25 years).

I%= 3.6

PV = 400 000

PMT = – 2024

FV = 0

P/Y = 12

C/Y = 12

Question 21

There are four statements to consider:

1 An effective interest rate is the same as a nominal interest rate if interest compounds annually.

2 Effective interest rates increase as the number of compounding periods per year increases.

3 A nominal rate of 12% per annum is equivalent to a nominal rate of 1% per month

4 An effective interest rate can be lower than a nominal interest rate.

Statements 1, 2 and 3 are true.

Question 23

Li's investment will amount to \$4838.60 after 5 years ($4000(1 + 0.0388)^5$).

The recurrence relation $J_0 = 3500$, $J_{n+1} = J_n + 267.72$ will also amount to \$4838.60 after 5 years.

$(3500 + 267.72 \times 5)$.

Question 24

Three steps are required.

Step 1: Determine the annual interest rate.

$0.52\% \times 4 = 2.08\%$ p.a.

Step 2: Consider the recurrence relation.

$D_1 = C$, $D_{n+1} = D_n$

This statement tells us that the additional amount per quarter is constant.

Step 3: Use Finance Solver.

$N = 2 \times 4$

I% = 2.08

PV = –10 500

PMT = SOLVE = 215.55…

FV = 12 700.95

P/Y = 4

C/Y = 4

Section B – Modules

Module 1 – Matrices

Question	Correct answer	% A	% B	% C	% D	% E
1	E	2	1	8	6	83
2	D	2	24	6	68	0

Question	Correct answer	% A	% B	% C	% D	% E
3	C	1	3	71	2	22
4	C	5	2	87	4	1
5	D	13	6	5	71	4
6	E	9	11	5	43	31
7	A	42	12	25	14	7
8	A	31	29	7	16	16

Students did not score as highly on questions involving the use of a matrix equations (Questions 6, 7 and 8).

Question 6

The system of simultaneous equations given could be represented as a matrix equation:

$$\begin{bmatrix} 0 & 1 & 1 \\ 1 & -1 & 1 \\ -1 & 1 & 0 \end{bmatrix} \times \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 & 1 & 1 \\ 1 & -1 & 1 \\ -1 & 1 & 0 \end{bmatrix}^{-1} \times \begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 & -1 & -2 \\ 1 & -1 & -1 \\ 0 & 1 & 1 \end{bmatrix} \times \begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}$$

Question 7

The product *KM* will result in a column matrix *P*.

The first row in matrix *K* will 'move' the element in the third row of matrix *M* to the first row of matrix *P*.

Question 8

A calculation that determines the total time that it would take each of Henry, Irvine or Jean must generate a 3×1 or a 1×3 matrix.

Option A is correct.

Option B finds the total time for the four different departments.

Option C finds the time for each of Henry, Irvine or Jean to service the laptops separately from the desktops.

Option D finds the time for each of Henry, Irvine or Jean to service equipment at the four different departments.

Option E finds the total time altogether.

Module 2 – Networks and decision mathematics

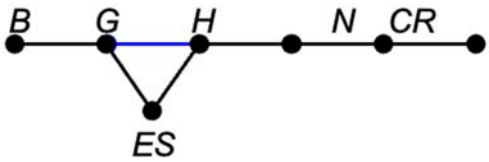
Question	Correct answer	% A	% B	% C	% D	% E
1	E	6	9	3	2	80
2	C	2	28	49	12	9
3	D	8	27	18	39	6
4	A	21	22	37	18	2
5	E	9	8	21	6	54
6	C	20	25	48	4	2
7	B	9	49	17	12	13
8	D	8	20	23	34	13

Students did not score as highly on questions that included critical path analysis (Question 6, 7 and 8) and questions interpreting diagrams, tables or graphs (Questions 2, 3 and 4).

Question 2

Inspection of the map shows Belize shares a border with Guatemala, Guatemala shares a border with Honduras and El Salvador, El Salvador shares a border with Honduras, Honduras shares a border with Nicaragua, Nicaragua shares a border with Costa Rica and Costa Rica shares a border with Panama. This is seven borders; therefore, the network diagram will have seven edges.

A diagram can help, such as :



Question 3

The requirement in this question was to find the allocation that maximised the total distance.

Option A total distance = 59.6

Option B total distance = 61.6

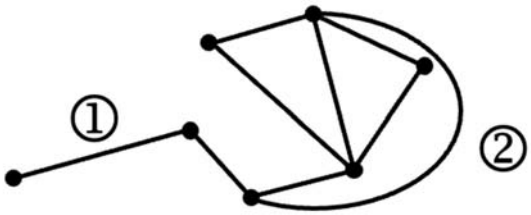
Option C total distance = 60.4

Option D total distance = 61.8

Option E total distance = 60.8

Question 4

The given graph is planar. It can be redrawn with no crossing edges.



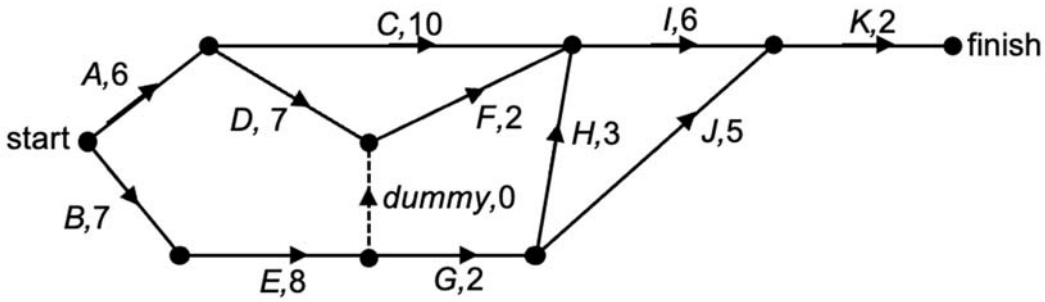
Question 6

The critical path for the project is *CEGIL* (35 hours).

Activity *G* cannot start until after activity *E* is completed.

Question 7

Consider either the table or the network diagram.



Activity *F* starts after the completion of activities *D* and *E*, activity *G* start after activity *E* only.

Question 8

The critical path is *BEGHIK.*

Inspection of the activities **not** on the critical path show that all other activities have float times as shown in the table below.

Activity	*A*	*C*	*D*	*F*	*J*
Float time	4	4	5	3	4

The sum of these float times is 20.

Module 3 – Geometry and measurement

Question	Correct answer	% A	% B	% C	% D	% E
1	D	2	1	1	93	3
2	D	6	4	3	81	5
3	B	7	67	14	7	4
4	C	6	8	49	32	4
5	C	6	11	24	53	5
6	E	17	8	18	16	38
7	B	4	69	19	6	1
8	C	29	19	40	7	4

Students did not score as highly on questions that required the use of the sine or cosine rule (Questions 6 and 8) or the question on time zones (Question 4).

Question 4

The three cities in order of longitude are Williamsburg (37ºN, 77ºW), Mountain Grove (37ºN, 92ºW) and Santa Cruz (37ºN, 122ºW).

The time difference between Williamsburg and Mountain Grove is one hour, the time difference between Mountain Grove and Santa Cruz is two hours and the time difference between Williamsburg and Santa Cruz is three hours.

The sun will rise in Williamsburg first, one hour later it will rise in Mountain Grove and two hours later it will rise in Santa Cruz.

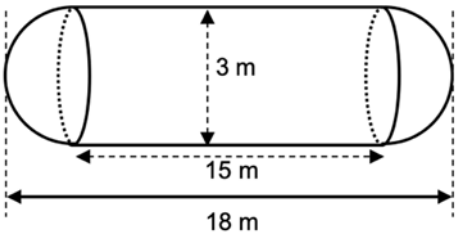
At 3 p.m. in Santa Cruz it will be 5 p.m. in Mountain Grove and 6 p.m. in Williamsburg.

Question 5

This question could be solved using a ratio.

The storage tank consists of a cylinder and two hemispherical ends.

The total length of the storage tank is 15 m + 2 × 1.5 m = 18 m.



Model : Actual

D : 3

300 : 18

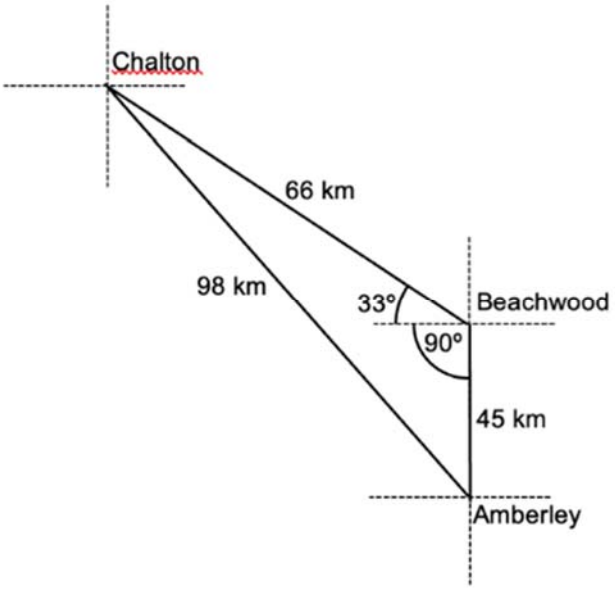
Solve $\frac{D}{300} = \frac{3}{18}$ gives D = 50 mm

Question 6

A diagram is helpful.

Angle at Beachwood = 90º + (303º – 270º) = 123º

Using the sine rule $\frac{\sin(C^\circ)}{45} = \frac{\sin(123^\circ)}{98}$, gives $C = 22.65^\circ$



The bearing from C to A = 123º + 22.65º = 145.65º ≈ 146º

Question 8

Distance from the camera to the foot of the ramp = $\frac{20}{\cos(90?\ \ 35?}$ = 34.869…

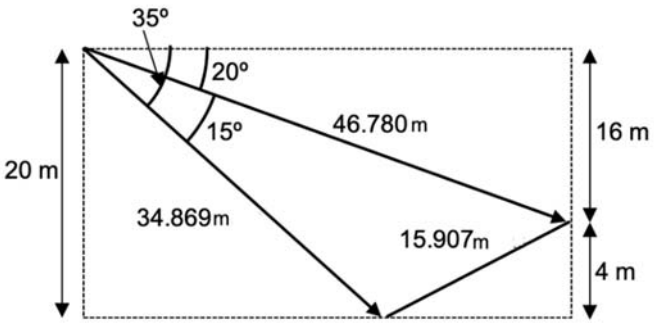
Distance from the camera to top of the ramp = $\frac{20-4}{\sin 20^\circ}$ = 46.780…

Angle between lines to ramp = 35º – 20º = 15º

Length of ramp from use of Cosine Rule with 34.869…, 46.780…, 15º

$r^2 = 46.781^2 + 34.869^2 - 2(46.781)(34.869)(\cos(15^\circ))$

Ramp length = 15.907…



Module 4 – Graphs and relations

Question	Correct answer	% A	% B	% C	% D	% E
1	E	2	5	1	1	90
2	E	1	3	9	24	62
3	A	78	8	9	2	3
4	C	4	9	73	10	3
5	B	5	73	7	11	3
6	B	16	43	15	14	12
7	A	52	8	15	18	6
8	D	11	10	24	29	25

Students did not score as highly on the inequality question (Question 6) or on the linear programming question (Question 8).

Question 6

Three drops of white paint (x) ≥ Two drops of red paint (y) $x \geq \frac{3}{2}y$

Check, when y = 2, $x \geq \frac{3}{2} \times 2$, x ≥ 3.

Question 8

A If $\frac{a}{b} = 2$, the maximum will occur at six integer values not eight. (FALSE)

B Since $b > 0$ the point (3, 12) which is in the feasible region, will always result in a higher Z value than (3, 10). As would the point (3, 11). Therefore (3, 10) will never maximise Z. (FALSE)

C If $\frac{a}{b} > 2$, the maximum will occur at (7, 4) not (2, 14) (FALSE)

D If $\frac{a}{b} > 6$, the minimum will occur at (2, 12) (TRUE)

E The minimum will occur at (3, 6) if $\frac{2}{3} \leq \frac{a}{b} \leq 6$, see below.

If $\frac{a}{b} = 6$, the minimum will also occur at (2, 12).

If $\frac{a}{b} = \frac{2}{3}$, the minimum will also occur at (6, 4).

If $0 < \frac{a}{b} < \frac{2}{3}$ the minimum will occur at only (6, 4).

If $\frac{a}{b} > 6$, the minimum will occur at only (2, 12). (FALSE)