2021 VCE Further Mathematics 2 external assessment report

General comments

Students were required to complete:

- a compulsory core section of Data analysis (worth 24 marks)
- a compulsory core section of Recursion and financial modelling (worth 12 marks)
- two selected modules (worth 12 marks each).

The percentage of students selecting each module in 2021 is shown in the following table.

Module	% 2021
Matrices	44.8
Networks and decision mathematics	28
Geometry and measurement	12
Graphs and relations	13.7

Scanned images are used for assessing and students should ensure their answers can be clearly read. Students are urged to take great care with the presentation of their responses. They should ensure their responses are written in a dark colour (e.g. black non-erasable pen or 2B pencil) so they are readable when scanned.

A formula sheet is provided with the examination. Students should familiarise themselves with the formula sheet during reading time, or the formula sheet published on the VCAA website well before the examination.

It is recommended that when students complete practice papers at home they allow 15 minutes reading time first. Students may not always use the reading time efficiently or effectively. Practising with this time will assist greatly as it simulates the actual exam experience.

After completing a question, students should read the question again to ensure that they are answering appropriately and that the requirements of the question have been met. For example in core Question 7c., students were asked to write a rule for the value of a coffee machine. Since an earlier question had related value in terms of time, many students failed to recognise the change to value in terms of number of cups produced.

Transcription errors can be costly. Students need to take great care when transferring their answers from the calculator. It was not uncommon to see answers with a 0 or 1 missing or digits interchanged.

Many questions on the examination were worth one mark only and with these questions the mark was awarded for a correct answer. For all questions worth more than one mark, students are strongly advised to show working. An incorrect answer on its own will not be awarded any marks in a two-mark question; however, sometimes a method mark can be awarded. For example in core Question 4c., a method mark was available for a student who correctly found the theoretical year of approximately 2054.176 but who was then unable to recognise that the answer needed was the next Olympic year of 2056.

When descriptive answers are required to a question, students are strongly advised to keep answers brief. An answer in point form is acceptable.

Sometimes a mark is available for students who simply know definitions or have written these in their bound reference and can apply the definition within the context of the question. For example in core Question 6b., the definition of a perpetuity is that its value does not change over time. This accessible question was incorrectly answered by many students.

Questions that were not well answered often involved the instruction to 'show that' a particular answer could be obtained. Students must work towards the given result with all relevant steps shown. When a student started with the answer given, the mark could not be awarded. For example, in core Question 8b., students were asked to **show that** the compound interest rate for a loan was 2.6% per annum. A student who worked back from this value to find the factor of 1.001 in the recurrence relation could not be given the mark.

Students should bring a ruler to accurately draw straight lines. These lines are often required in both Data analysis and Graphs and relations.

In questions where no instruction to round is given, an exact answer is required as rounding does not apply. For example in Geometry Question 1c., the required answer was 100.86 square centimetres; as rounding did not apply, an answer such as 101 could not be accepted.

Rounding of answers to a specified type and level of accuracy is an important skill that students are required to demonstrate. It was clear that when writing the equation of a least squares line, many students did not understand the difference between decimal places rounding and significant figures rounding.

In Recursion and financial modelling, students are often instructed to answer questions correct to the nearest cent (two decimal places). For example, in core Question 9b., the required answer to the nearest cent was $7039.20. A student who answered $7039.2 could not be awarded the mark. This given answer actually represents one decimal accuracy of any value between $7039.15 and $7039.24.

In core questions where a maximum of one rounding error was penalised, a second rounding error would not have resulted in the loss of marks if either:

- a correct calculation was shown prior to the final incorrectly rounded answer
- additional correct decimal places could be seen earlier in the response or in the final answer.

Specific information

Note: This report provides sample answers or an indication of what answers may have included.

The statistics in this report may be subject to rounding resulting in a total more or less than 100 per cent.

Core – Data analysis

Question 1a.

Mark	0	1	Average
%	48	52	0.5

The correct answer was 3 variables.

This question was not answered well. A common error was counting the number of values (45) instead of variables.

Question 1b.

Mark	0	1	Average
%	13	87	0.9

The correct answer was 1.81

This question was answered well.

Question 1c.

Mark	0	1	Average
%	22	78	0.8

The correct answer was 0.5

This question was answered well. A few responses did not round as required.

Question 1d.

Mark	0	1	Average
%	40	60	0.6

The correct answer was 84%.

The most common incorrect answer was 16%.

Question 1e.

Mark	0	1	Average
%	53	47	0.5

Maximum value = Q_3

Many responses mentioned that the maximum was 1.87 without indicating this was also Q_3. Students who used the term 'upper fence' appeared to think that this was the end of the box.

Question 1f.

Mark	0	1	2	Average
%	49	42	9	0.6

The correct answer was 45.89 m.

Most students who identified the correct quartiles gave a response of 45.88 m, not recognising that potential medal winners threw greater than that distance.

Question 2a.

Mark	0	1	Average
%	48	52	0.5

The correct numbers were 0.0393 and 5.28

Many students rounded to three decimal places rather than three significant figures.

Question 2b.

Mark	0	1	2	Average
%	67	3	29	0.6

The correct answer was 38%.

Many students struggled with the calculation of Pearson's correlation coefficient from summary statistics. Some students did not attempt this question.

Question 3a.

Mark	0	1	Average
%	10	90	0.9

The correct answer was year.

Most students answered this correctly.

Question 3b.

Mark	0	1	Average
%	81	19	0.2

The correct answer was –0.938

Many students found 0.938 but left off the negative sign.

Question 3c.

Mark	0	1	Average
%	68	32	0.3

The correct answer was 0.1515

The instruction to 'write down' the average decrease suggested no calculation was required. Some calculated the average decrease using the points (1960, 60) and (2000, 54), giving the less accurate 0.15.

Question 3d.

Mark	0	1	Average
%	32	68	0.7

The correct answer was –0.27

Some students did not write the negative sign.

Question 3ei.

Mark	0	1	Average
%	15	85	0.9

The correct answer was $357.1 - 0.1515 \times 2032$

This question was answered well with the substitution clearly shown.

Question 3eii.

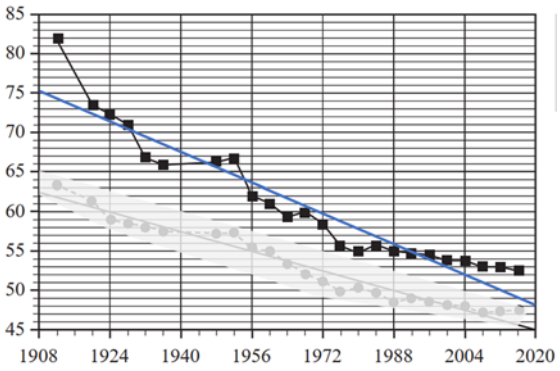
Mark	0	1	Average
%	82	18	0.2

Same decreasing trend continues in the future.

Students found it hard to convey the idea of the trend continuing. It was common to see reference to extrapolation or linearity but without linking it to the given equation.

Question 4a.

Mark	0	1	Average
%	54	46	0.5



Students who wrote the endpoints of (1908,75.256) and (2020,48.04) on the graph tended to be successful. The line needed to be positioned correctly on the graph. The best way to achieve this would be to establish the two endpoints and connect the two using a ruler. While the writing of coordinates is not necessary to achieve this aim, it is useful.

Many students left the question out entirely.

Question 4b.

Mark	0	1	2	Average
%	20	12	67	1.5

The correct answer was 2.7 seconds.

This was generally answered well but rounding too soon from calculations was common.

Question 4c.

Mark	0	1	2	Average
%	57	4	40	0.9

The correct answer was 2056

Only a small proportion of students successfully solved the two equations. Many used a trial approach and substituted in numerous Olympic years.

Question 5a.

Mark	0	1	2	Average
%	65	19	16	0.5

$$\frac{1}{difference} = -2.234 + 0.001209 \times year$$

Not many students obtained two marks. Many students who used the correct transformation rounded values incorrectly or wrote the variables incorrectly.

Question 5b.

Mark	0	1	Average
%	86	14	0.2

The correct answer was 4.5 seconds.

A common response was 0.2 gained by using the correct coefficients but using difference rather than the reciprocal of difference.

Core – Recursion and financial modelling

Question 6a.

Mark	0	1	Average
%	41	59	0.6

The correct answer was $22 680

This was often answered incorrectly.

Question 6b.

Mark	0	1	Average
%	53	47	0.5

The correct answer was $420 000

The definition of a perpetuity continues to be misunderstood by many students.

Question 6c.

Mark	0	1	Average
%	62	38	0.4

The correct numbers were 420 000 and 1.0045

A factor of 1.054 was a common incorrect response.

Question 7a.

Mark	0	1	Average
%	46	54	0.6

The correct answer was 28 800

Question 7b.

Mark	0	1	Average
%	51	49	0.5

The correct answer was 12%.

Question 7c.

Mark	0	1	Average
%	79	21	0.2

The correct numbers were 12 000 and -0.05

This question was not answered well.

Many students did not recognise that the value was of the machine in terms of the number of cups of coffee made.

Question 8a.

Mark	0	1	Average
%	32	68	0.7

The correct answer was $569 377

This question was answered reasonably well.

Question 8b.

Mark	0	1	Average
%	73	27	0.3

The correct answer was $(1.001 - 1) \times 26 \times 100$

Some students did not recognise the detail required and wrote an equation involving r and then 'Solve'.

Students must be aware that any response including CAS syntax/notation cannot attract full marks. In the instance of 1-mark questions (particularly 'show that' questions such as this one), the student may not be awarded the mark.

Question 8c.

Mark	0	1	Average
%	85	15	0.2

The correct answer was $1198.59

A common incorrect response was $1197.39. This was the balance before the last payment but does not include the interest to be added to the outstanding balance for the last payment.

Question 9a.

Mark	0	1	2	Average
%	58	21	21	0.7

V_0 = \$152 431

V_1 = 1.00425 × 152 431 – 900 = \$152 178.83

V_2 = 1.00425 × 152 178.83 – 900 = \$151 925.59

Many students correctly calculated the factor of 1.00425 but either added the \$900 or did not show the recursive calculations in full.

Question 9b.

Mark	0	1	Average
%	92	8	0.1

The correct answer was \$7039.20

Finance solver entries to firstly find the future value after two years:

N = 24

I% = 5.1

PV = –152431

PMT = 900

FV = **146 073.7405**

P/Y & C/Y = 12

Finance solver entries to then find the balance required to provide 276 remaining payments at the new interest rate:

N = 276

I% = 4.6

PV = **–153 112.9399**

PMT = 900

FV = 0

P/Y & C/Y = 12

Extra amount to be added = 153 112.94 – 146 073.74 = 7039.20

Module 1 – Matrices

Question 1a.

Mark	0	1	Average
%	7	93	1.0

The correct answer was 3×1

This question was answered well.

Question 1b.

Mark	0	1	Average
%	52	48	0.5

The correct answer was 1.05

This question was not answered well. Students often responded with 0.05

Question 2a.

Mark	0	1	Average
%	8	92	0.9

The correct answer was Brie and Dex.

Question 2b.

Mark	0	1	Average
%	17	83	0.9

The correct answer was Elena – Dex – Brie – Chai.

This question was answered well.

Question 2c.

Mark	0	1	Average
%	79	21	0.2

The correct answer was Alex – Brie – Dex and Alex – Elena – Dex.

This question was not answered well. Most students appeared not to interpret this question correctly.

Question 3a.

Mark	0	1	Average
%	72	28	0.3

The correct answer was 1160

$0.15 \times 3200 + 0.2 \times 2000 + 0.1 \times 2800$

Many students made little progress on this question.

Question 3b.

Mark	0	1	Average
%	22	78	0.8

$$\begin{bmatrix} 3060 \\ 1900 \\ 3040 \end{bmatrix}$$

This question was answered well.

Question 3c.

Mark	0	1	Average
%	83	17	0.2

The correct answer was 89%.

$$\frac{0.85 \times 3200}{3060}$$

This question was not answered well.

85% taken directly from the transition matrix was a very common incorrect answer.

Question 3d.

Mark	0	1	Average
%	78	22	0.2

$T^n \times S_0 \approx \begin{bmatrix} 2400 \\ 1600 \\ 4000 \end{bmatrix}$ and $\frac{4000}{8000} = 50\%$ $(n \geq 35)$

Many students found a long-term matrix but were unsure what to do with it to answer the question.

Question 3e.

Mark	0	1	Average
%	85	15	0.2

The correct answer was 200

This question was not answered well.

Question 4

Mark	0	1	2	Average
%	70	20	10	0.4

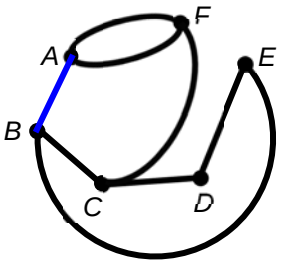
$$\begin{array}{c} \\ I \\ J \\ K \\ L \\ M \end{array} \begin{array}{c} \begin{array}{ccccc} I & J & K & L & M \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 \end{bmatrix} \end{array}$$

Only some students were successful in filling in the entire matrix.

Module 2 – Networks and decision mathematics

Question 1a.

Mark	0	1	Average
%	7	93	1.0



This question was answered well.

Question 1b.

Mark	0	1	Average
%	12	88	0.9

The correct answer was 2

This question was answered well.

Question 1ci.

Mark	0	1	Average
%	8	92	0.9

A – *B* – *E* – *D* – *C* or *C* – *D* – *E* – *B* – *A* (the reverse)

This question was answered well.

Question 1cii.

Mark	0	1	Average
%	40	60	0.6

The correct answer was Hamiltonian cycle.

A proportion of students just wrote ‘cycle’, which was not accepted.

Question 2a.

Mark	0	1	Average
%	15	85	0.9

The correct answer was 86 km.

This question was answered well.

A small number of students wrote the correct travel sequence but did not indicate the shortest distance as required by the question.

Question 2b.

Mark	0	1	Average
%	35	65	0.7

The correct answer was *K.*

This question was answered reasonably well.

Question 3a.

Mark	0	1	Average
%	76	24	0.3

The correct answer was 1330

This question was not answered well.

Many students did not demonstrate they had tried to find the minimum cut.

Question 3bi.

Mark	0	1	Average
%	62	38	0.4

The correct answer was *A* to *D.*

Question 3bii.

Mark	0	1	Average
%	94	6	0.1

The correct answer was 780

This question was not answered well.

Question 4a.

Mark	0	1	Average
%	54	46	0.5

The correct answer was 14

This question was answered reasonably well.

Question 4b.

Mark	0	1	Average
%	78	22	0.2

The correct answer was 7

This question was not answered well. Many students did not seem to realise there were two critical paths.

Question 4c.

Mark	0	1	Average
%	91	9	0.1

The correct answer was $380 000

This question was not answered well. Few students recognised the need to reduce *A* by one week and *L* by two weeks.

Module 3 – Geometry and measurement

Question 1a.

Mark	0	1	Average
%	13	87	0.9

$\frac{4}{3} \times \pi \times 2^3$

This question was answered well.

Question 1b.

Mark	0	1	Average
%	26	74	0.8

The correct answer was 35.41 cm^3

This question was generally answered well.

Question 1c.

Mark	0	1	Average
%	19	81	0.8

The correct answer was 100.86 cm^2

This question was answered reasonably well.

Question 1d.

Mark	0	1	Average
%	57	43	0.5

The correct answer was 24

26 was a very common incorrect answer gained from dividing volume of the shelf by the volume of a box.

Question 2a.

Mark	0	1	Average
%	49	51	0.5

The correct answer was 32.66 cm^2

$\frac{1}{2}(4.57 + 2.13) \times 9.75$

Some students did not use efficient methods.

Question 2b.

Mark	0	1	Average
%	52	48	0.5

The correct answer was 26.5 m.

This question was answered reasonably well.

Question 2c.

Mark	0	1	Average
%	58	42	0.4

Distance = $\sqrt{2.7^2 + 3.1^2 - 2 \times 2.7 \times 3.1 \times \cos 119°}$

Students needed to show the correct substitution and the square root sign.

Question 3a.

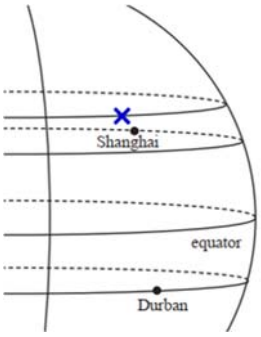
Mark	0	1	Average
%	30	70	0.7

The correct answer was Wei-Yi and Ozlem.

This question was generally answered well. Some students gave only one name.

Question 3b.

Mark	0	1	Average
%	59	41	0.4



Many students did not select the correct parallel on which to place the cross. Others chose the correct parallel but placed the cross on the dotted side.

Question 3ci.

Mark	0	1	Average
%	59	41	0.4

The correct answer was $r = 6400 \cos 41^\circ$.

This question was not answered well.

Question 3cii.

Mark	0	1	Average
%	70	30	0.3

The correct answer was 8683 km.

Many students did not attempt this question.

Question 3d.

Mark	0	1	Average
%	84	16	0.2

The correct answer was 10 hours.

This question was not answered well.

Module 4 – Graphs and relations

Question 1a.

Mark	0	1	Average
%	27	73	0.8

The correct answer was 4 minutes.

Question 1b.

Mark	0	1	Average
%	32	67	0.7

The correct answer was 100 m/min.

Question 2a.

Mark	0	1	Average
%	17	83	0.9

The correct answer was 6

This question was answered reasonably well.

Question 2b.

Mark	0	1	Average
%	63	37	0.4

The correct answer was 3, 6, 11, 15, 20

Many students were able to identify some, but not all, of the required values. All values were required for the mark to be awarded.

Question 2c.

Mark	0	1	Average
%	79	21	0.2

$y \leq \frac{3}{5}x$

Consider a ratio of 3 blocks to 5 blocks for John to Christy.

$\begin{matrix} y:x \\ 3:5 \end{matrix}$ and 'at most' is represented by $\leq$

An inequality is $\frac{y}{x} \leq \frac{3}{5}$ which is equivalent to $y \leq \frac{3}{5}x$

This question was not answered well.

Question 3a.

Mark	0	1	Average
%	71	29	0.3

$$k = \frac{92500 - 10000}{15} = 5500$$

Question 3b.

Mark	0	1	Average
%	54	46	0.5

The correct answer was 10

This question was answered reasonably well.

Question 3c.

Mark	0	1	Average
%	83	17	0.2

The correct numbers were 1000 and $-10\,000$

This question was not answered well.

Question 4a.

Mark	0	1	Average
%	57	43	0.5

The total time for all sessions each day cannot exceed 600 minutes.

Some students confused time with number of sessions. Others simply said the time must be less than 600 minutes.

Question 4b.

Mark	0	1	Average
%	57	43	0.5

The correct answer was 8

8.5 was quite often given by students who didn't recognise that an integer number of sessions was required.

Question 4ci.

Mark	0	1	Average
%	57	43	0.5

The correct answer was $900

This question was answered reasonably well.

Question 4cii.

Mark	0	1	Average
%	80	20	0.2

The correct answer was (0, 15), (4, 12), (8, 9)

$P = 45x + 60y$

Slope of objective function of $-\frac{3}{4}$ equals the slope of $30x + 40y = 600$

Therefore maximum occurs at all integer points on the line segment.

Many students were able to identify the two extreme points that gave maximum profit, but did not recognise there was a third solution on the line segment.