VICTORIAN CURRICULUM AND ASSESSMENT AUTHORITY

2018 VCE Further Mathematics 1 examination report

General comments

Students found the majority of questions accessible in the Further Mathematics examination 1 in 2018. Students found some questions involving application of the key skills and key knowledge from the study design challenging, such as Questions 20, 22 and 23 in the Core section.

Specific information

The tables below indicate the percentage of students who chose each option. The correct answers are indicated by shading.

The statistics in this report may be subject to rounding resulting in a total more or less than 100 per cent.

Section A – Core

In 2018, the Core section comprised two components: Data analysis (Questions 1–16) and Recursion and financial modelling (Questions 17–24).

Question	% A	% B	% C	% D	% E	% No answer
1	1	5	85	7	3	0
2	85	8	4	2	1	0
3	86	5	4	2	3	0
4	2	4	12	14	68	0
5	9	69	10	6	5	0
6	6	16	3	71	5	0
7	5	73	8	10	4	0
8	16	25	8	46	5	0
9	45	4	8	4	38	0
10	20	51	9	6	14	0
11	58	11	21	6	4	1
12	15	22	5	4	54	0
13	9	13	22	49	6	2
14	19	13	42	13	13	1
15	4	80	8	6	2	0
16	10	11	51	19	8	1
17	1	3	5	79	11	0
18	5	17	61	11	5	0
19	28	4	6	54	7	1
20	9	39	9	28	14	0
21	10	59	13	6	10	1

Question	% A	% B	% C	% D	% E	% No answer
22	10	10	9	41	29	1
23	29	15	37	9	9	1
24	9	13	17	15	45	1

Data analysis

Students generally answered the questions in this section well, particularly questions that required standard, routine calculations (Questions 1, 2, 3 and 15). Questions that required the use or analysis of graphical or tabular information were answered less well (Questions 8, 9, 13 and 16).

Question 8

Students were asked to identify the equation of the least squares line drawn on a graph that contained 16 points. While it was not possible to determine exact slope and intercept values from the graph, students should have been able to approximate these values. Many students incorrectly assumed that the intercept value of the line was 67.2, read directly from the graph; however, this is only possible if the horizontal axis begins at value zero. Students are encouraged to look carefully at graphs before choosing what might seem to be the obvious answer.

Question 9

The value of the correlation coefficient *r* was required, given that the coefficient of determination was $r^2 = 0.8339$. Many students incorrectly took the positive square root value and chose option E.

The scatterplot on the examination showed that the direction of the association was negative and therefore -0.913 was required.

Question 13

Students needed to be familiar with the rules required for finding a least squares line equation from summary statistics.

The slope of the line = $r\dfrac{s_y}{s_x}$

Solving $1.31 = r \times \dfrac{3.24}{2.33}$ gives $r = 0.94$, rounded to two decimal places.

Question 14

Students needed to be aware that reversing the two variables will give a different equation.

The slope and intercept will therefore both change in value, the predictions the line gives will change and hence the residual values will also change.

The correlation coefficient will not change in value as the degree of scattering of the points remains unchanged – i.e. the scattering of ‘*y*’ values relative to ‘*x*’ is the same as the scattering of ‘*x*’ values relative to ‘*y*’.

Question 16

Students firstly needed to determine the quarterly averages for 2016 and 2017:

quarterly average for 2016 = $\dfrac{1.73+2.87+3.34+1.23}{4} = 2.2925$

quarterly average for 2017 = $\dfrac{1.03+2.45+2.05+0.78}{4} = 1.5775$

The seasonal index for Quarter 3 is the average of the Quarter 3 indices for each year.

$\dfrac{\dfrac{3.34}{2.2925}+\dfrac{2.05}{1.5775}}{2}=1.38$, rounded to two decimal places.

Recursion and financial modelling

The questions students found most challenging involved the comparison of different financial situations (Questions 19 and 20) or more complicated calculations (Questions 22, 23 and 24).

Question 19

This question required an understanding that both the interest rate and compounding period in combination determine the overall cost of the loan. Students who selected option A seemed to simply select the lowest interest rate without considering the compounding period.

Question 20

Students needed to recognise that the rate of reduction in value is constant in both phases of depreciation, indicating flat-rate depreciation, not reducing balance depreciation.

Question 22

This question required the use of a finance solver application and could have been solved by determining the future value of the loan after five years, using the following entries:

N = 60 (5 years)

I% = 3.72

PV = 175260.56

PMT = –3200 FV = ?

P/Y = 12

C/Y = 12

to result in a future value of –$368.12

The negative indicates that this amount must still be paid back to the bank and added to the regular payment of $3200 to give $3568.12

Students who chose option D did not add the interest that was required to be paid with the final payment.

Question 23

This question required the analysis of an amortisation table and the calculation of two interest rates.

Starting interest rate = $\dfrac{967.08}{230256.78}\times 100=0.42\%$ per month

New interest rate = $\dfrac{1002.26}{227785.76}\times 100=0.44\%$ per month

Increase of 0.02% per month $\times 12$ = 0.24% per annum

Question 24

This question required the use of a finance solver application. Some students were able to do so successfully, although it seemed that problems involving changing conditions were particularly challenging for some students.

This question could have been solved by firstly determining the future value of the annuity after the first 2 years, using the following entries:

N = 24 (2 years)

I% = 3.24

PV = –265 298.48

PMT = –1000 FV = ?

P/Y = 12

C/Y = 12

to result in a future value of $307 794.50 to the nearest cent

The second stage of the solution required the determination of the new monthly payment using the following entries:

N = 96 (8 years)

I% = 3.20

PV = –307 794.50 (Entered as negative here. It represents the new principal value invested.)

PMT = ?

FV = 600 000

P/Y = 12

C/Y = 12

to result in a PMT value of –1854.05, meaning outgoing payments are close to $1854 and the correct answer of option E.

Understanding of the sign convention for the finance solver is very important, as is the careful tracking of values used in subsequent calculations.

Section B – Modules

Students were required to complete questions from two of the four modules.

The selection of modules by students in 2018 is shown is shown in the table below.

Module	% 2018
Matrices	89
Networks and decision mathematics	50
Geometry and measurement	29
Graphs and relations	32

Module 1 – Matrices

Question	% A	% B	% C	% D	% E	% No Answer
1	9	7	3	78	2	0
2	5	15	2	10	68	0
3	78	6	6	6	3	1
4	5	69	9	9	6	1
5	14	42	15	8	18	2
6	6	8	81	3	3	0
7	21	14	16	20	28	1
8	12	46	13	18	11	1

The questions that students found most challenging involved the solving of simultaneous equations in matrix form (Question 5), the application of a recurrence relation (Question 7) and the long-term expectation from a transition matrix (Question 8).

Question 5

Simultaneous equations in matrix form can be set up directly from the table as follows:

$$\begin{bmatrix} 5 & 7 & 6 & 8 \\ 8 & 6 & 9 & 7 \\ 7 & 8 & 7 & 6 \\ 8 & 8 & 5 & 5 \end{bmatrix} \begin{bmatrix} c \\ r \\ s \\ w \end{bmatrix} = \begin{bmatrix} 160 \\ 172 \\ 165 \\ 162 \end{bmatrix}$$

To solve

$$\begin{bmatrix} c \\ r \\ s \\ w \end{bmatrix} = \begin{bmatrix} 5 & 7 & 6 & 8 \\ 8 & 6 & 9 & 7 \\ 7 & 8 & 7 & 6 \\ 8 & 8 & 5 & 5 \end{bmatrix}^{-1} \begin{bmatrix} 160 \\ 172 \\ 165 \\ 162 \end{bmatrix}$$

$$\begin{bmatrix} c \\ r \\ s \\ w \end{bmatrix} = \begin{bmatrix} 8 \\ 6 \\ 1 \\ 9 \end{bmatrix}$$

Question 7

This question involved finding a state matrix, A_2, given the following state matrix A_3.

Using the matrix recurrence rule

$$A_{n+1} = TA_n - D$$

$$A_3 = TA_2 - D$$

$$TA_2 = A_3 + D$$

$$TA_2 = \begin{bmatrix} 1666 \\ 2850 \\ 2184 \end{bmatrix}$$

$$A_2 = T^{-1}\begin{bmatrix} 1666 \\ 2850 \\ 2184 \end{bmatrix} = \begin{bmatrix} 1630 \\ 2800 \\ 2270 \end{bmatrix}$$

Therefore, the number in the west is 2270 (option E).

Question 8

The long-term expectation involves raising the transition matrix to a high power, for example, 50.

The steady state matrix is independent of the initial state, so one suggested approach is to use **any** initial state matrix where the values added to 500, e.g. $\begin{bmatrix} 100 \\ 100 \\ 100 \\ 100 \\ 100 \end{bmatrix}$

$T^{50}\begin{bmatrix} 100 \\ 100 \\ 100 \\ 100 \\ 100 \end{bmatrix} = \begin{bmatrix} 49 \\ 96 \\ 124 \\ 151 \\ 80 \end{bmatrix}$ with values rounded to the nearest whole number.

Therefore, category K (option B) was the required answer.

Module 2 – Networks and decision mathematics

Question	% A	% B	% C	% D	% E	% No Answer
1	2	92	4	1	0	0
2	5	10	68	15	2	0
3	4	10	5	4	76	1
4	7	2	2	88	1	0
5	27	43	11	6	11	2
6	9	20	6	41	23	1
7	14	16	49	9	11	1
8	6	11	24	24	34	1

The questions students found most challenging involved critical path analysis (Questions 5 and 7), the understanding of planar graphs (Question 6) and the allocation of tasks for minimum completion time (Question 8).

Question 5

The activities that could be delayed are those that are not on the critical path.

There are two critical paths for this network (*A-D-H-K* and *B-F-J-K)* both of duration 15 weeks.

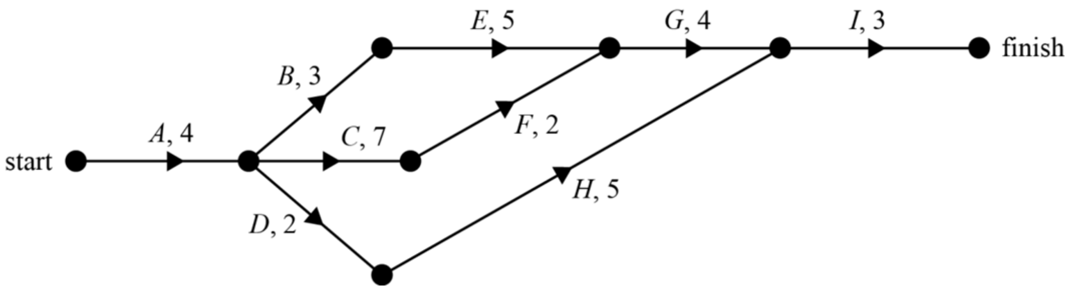
This leaves four other activities that have a float time, namely *C*, *E*, *G* and *I*.

Question 6

Of the five graphs presented, only option D could not be redrawn without intersecting edges.

Many students would have found this by trial, but some would have recognised option D as a complete graph with five vertices. Any complete graph with five, or more, vertices is non-planar.

Question 7



By sketching the network as shown above or following the table of predecessors, three paths are possible from the start of activity *A* to the finish of activity *I*.

A-B-E-G-I 19 hours

A-C-F-G-I 20 hours

A-D-H-I 14 hours

The minimum completion time is the longest path from start to finish. Therefore, 20 hours (option C) was the correct answer.

Question 8

The allocation of tasks by the supervisor results in a completion time of 12 minutes.

From inspection (or the Hungarian algorithm) we can determine that if $k = 0$ the minimum completion time is 10 minutes and if $k = 1$ the minimum completion time is 11 minutes.

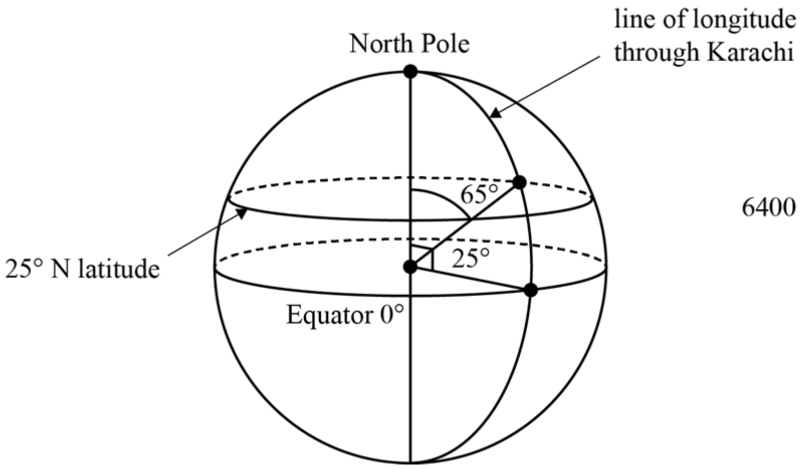
The first time the minimum completion time is 12 minutes is when $k = 2$ (the least value of k) and it remains 12 minutes as k increases. Options D and E were chosen by some students but these did not include this least value of 2.

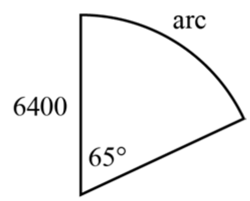
Module 3 – Geometry and measurement

Question	% A	% B	% C	% D	% E	% No Answer
1	3	92	3	0	2	0
2	80	3	6	8	1	1
3	5	26	43	16	9	1
4	10	9	24	13	42	2
5	8	32	51	5	4	1
6	10	17	41	21	9	2
7	14	66	8	8	3	1
8	5	10	25	51	8	1

Questions involving elementary application of the key skills from this module were answered correctly by the majority of students. The spherical geometry questions (Questions 3 and 5) and the applications involving bearings (Questions 4 and 6) caused difficulty for many students. Question 8 was also challenging for many students.

Question 3





The distance from any point on earth to the North Pole is found by using the angle between the radius through that point and the North Pole.

In this case the required angle was $90-25=65^{\circ}$ as shown on the diagram above.

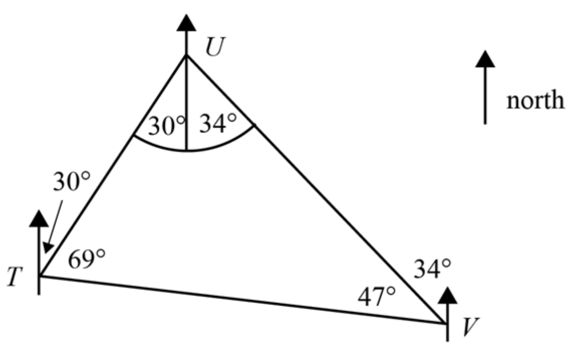
The required distance is along an arc with angle of 65° and radius of 6400 km.

The longitude $67°E$ was not relevant to this question.

Find the length of the arc using $l=r\times\frac{\pi}{180}\times\theta$

$l=6400\times\frac{\pi}{180}\times 65$, which is equivalent to option C.

Question 4



The third angle in the triangle is $180-(69+47)=64°$

As shown on the diagram, this is the sum of the alternate angle of $30°$ and $34°$

The bearing of U from V is the number of degrees clockwise from North, therefore $360-34=326°$

Question 5

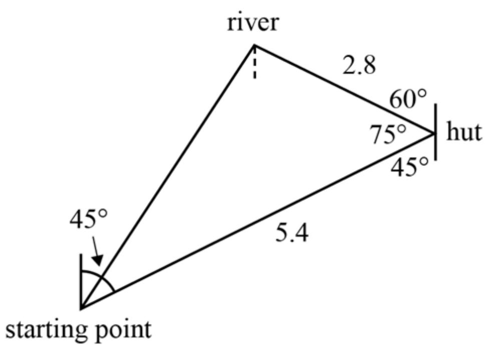
Students needed to recognise $15°$ of longitude equates to a one-hour time difference.

The difference in longitude between St Petersburg and Helsinki is $5°$.

$\frac{5}{15} \times 60$ = 20 minutes

St Petersburg is further east; therefore, the sun rises earlier, so 20 minutes later in Helsinki.

Question 6



From the diagram there are two sides and the included angle, therefore the cosine rule should have been used.

Distance from the starting point to the river $= \sqrt{2.8^2 + 5.4^2 - 2 \times 2.8 \times 5.4 \times \cos 75°}$ = 5.4 to one decimal place

The total distance is the perimeter of the triangle = 5.4 + 2.8 + 5.4 = 13.6 km

Question 8

Volume of cone $= \frac{1}{3}\pi r^2 h$

Solving $36 = \frac{1}{3}\pi \times 2.5^2 \times h$ gives $h = 5.5$ to one decimal place

By Pythagoras $x = \sqrt{2.5^2 + 5.5^2} = 6.04$ to two decimal places

The total surface area = $\pi r(r + x)$ = $\pi \times 2.5(2.5 + 6.04) = 67$ to the nearest whole number.

Module 4 – Graphs and relations

Question	% A	% B	% C	% D	% E	% No Answer
1	1	7	82	8	1	0
2	1	1	1	96	1	0
3	8	5	8	66	12	1
4	18	66	6	4	6	1
5	48	14	21	8	8	2
6	16	6	14	12	51	1
7	14	14	40	21	10	1
8	4	14	21	48	12	1

Many students were challenged by Questions 5, 6, 7 and 8, which included a higher level of analysis of the graphs and relations presented.

Question 5

The relationship is of the form $y = k \times \frac{1}{x^2}$

The gradient of the given line is 2 therefore $y = 2 \times \frac{1}{x^2}$

Therefore, $y = \frac{2}{x^2}$

Test the given point (1,2) for each option where y is plotted against x to see if this holds.

For option A, substituting $x = 1$ and $y = 2$ gives $2 = \frac{2}{1}$, which is a true statement.

Question 6

The cost of making 80 quilts is $800 + 80 \times 35 = \$3600$

Profit = Revenue – Cost

$1200 =$ Revenue – 3600

therefore, Revenue = $4800

Selling price = $\frac{4800}{80} = \$60$

Therefore, option E was correct.

Question 7

Since the objective function has its minimum at both point *M* and point *N*, then the gradient of the objective function must be the same as the gradient of the line containing these two points.

From the graph the gradient of the line from $(0,10)$ to $(5,0)$ is $\frac{0-10}{5-0} = -2$

A function of the form $Z = ax + by$ has gradient of $-\frac{a}{b}$

Testing each option:

A. gradient = $\frac{-1}{-2} = \frac{1}{2}$

B. gradient = $\frac{-1}{2} = -\frac{1}{2}$

C. gradient = $\frac{-2}{1} = -2$ therefore, this was the correct answer

D. gradient = $\frac{2}{1} = 2$

E. gradient = $\frac{-2}{2} = -1$

Question 8

Let *x* represent the time in minutes and *y* the distance in kilometres.

Since the fixed cost is $2.55, two simultaneous equations can be formed without this cost.

Judy: $8x+10y=14.20$

Pat: $20x+18y=27.80$

Solving gives $x=0.4$ and $y=1.1$

Cost for Roy will be $2.55+10\times0.4+15\times1.1$ = $23.05

Therefore, option D was correct.